Project 2: Vortex Panel Method

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1 Abstract

In this project, the vortex panel method was used to simulate flow conditions around a 2-dimensional *circular* cylinder by using panels of equal length with linearly varying strengths such that the solution is continuous from panel to panel. Particularly, the pressure distribution around a circular cylinder was evaluated using 32, 64, and 128 panels with the Kutta condition imposed at $\theta = 180°$ and $\theta = -150°$. The method of panels pressure distribution was compared to the pressure distribution found analytically using potential flow theory. Unsurprisingly, it was determined that the method of panels solution approached the analytical solution as the number of panels increased. Finally, the Thwaites boundary layer separation criterion was used to determine the separation points for both the symmetric and asymmetric cases.

2 Problem Statement

The main problem of this project was to develop a code base that can mathematically approximate a solid boundary of a shape, in this case a cylinder, as well as the flow around it using doublets arranged in linear panels. By importing different sets of coordinates, the same code should also be able to evaluate the flow around a non-cylindrical airfoil. Another essential issue was to simulate the viscous effect of air flowing smoothly off the trailing edge in an otherwise inherently inviscid approximation. Mathematically, this can be done by imposing a net circulation to force the separation point to the desired location; thus, the Kutta condition can be used with the method of panels by nulling vortex panel strengths at the trailing edge.

3 Formulation

The general formulation used was adapted from the vortex panel method presented in *Foundations of Aerodynamics* by Kuethe and Chow. The potential function for the combined field of the uniform flow and vortex panels is represented by:
$\psi(x,y) = Uxcos(\alpha) + Uysin(\alpha) + \sum_j \int_j \frac{\gamma s_j}{2\pi} tan^{-1}(\frac{y-y_i}{x-x_j}) ds_j$.
The vortex strength varies linearly along the length of the panels: $\gamma(s_j) = \gamma_j + (\gamma_{j+1} - \gamma_j)\frac{s_j}{S_j}$.
Since the normal velocity at the panel control points must be zero: $\frac{\partial\psi}{\partial n}|_i = 0$,
implementing the previous equations results in solving the following matrix equation to find γ_i:
$\sum_{j=1}^{m}(C_{n1_{ij}}\gamma_j + C_{n2_{ij}}\gamma_{j+1}) = 2\pi U(sin(\theta_i - \alpha))$ where the coefficients are defined in the Project Spec. The Kutta condition is incorporated with $\gamma_1' + \gamma_{m+1}' = 0$. After which, the tangential velocities and pressure coefficients are solved for at each panel's control point with the following:
$v_i = cos(\theta_i - \alpha) + \sum_{j=1}^{m+1} A_{ij}\gamma_j'$ and $c_{p_i} = 1 - v_i^2$.

To find the separation points, Thwaites boundarly layer separation criteria was used by determining which points along the cylinder's surface caused $K + 0.09$ to be at a minimum in Thwaites equation: $K = \frac{0.45}{U_e^6}\frac{dU_e}{dx}\int_0^x U_e^5(\xi)d\xi$

4 Implementation

Method: *Project2(fileName)*
The Project2(fileName) method served as the main and only MATLAB method in the code base. All the calculations were taken care of inside of this method. However; in hindsight, it would have been better to divide the code into more sections. The *fileName* input to this method is the file name of a .txt file with (X, Y) coordinates of an airfoil in the same format as .txt files downloaded from airfoiltools.com. The code then reads the .txt file into MATLAB matrices, and re-arranges it to a format that will work with the rest of my code. Next, I determine how many panels to use based on the number of boundary points given in the .txt file, and determined the midpoints of the boundary points to use as control points for the following calculations. Using the equations given in the Project Spec and the aforementioned Textbook, $C_{n1_{ij}}$, $C_{n2_{ij}}$, and the right hand side of the equation were found such that γ was solved for using MATLAB's backslash operator. γ was then used to find the tangent velocity at each control point and then the pressure coefficients using the equations mentioned in the Formulation section above. Changing the Kutta condition was done simply by altering the angle of attack of the cylinder. Once the pressure coefficient matrix was found, the theoretical pressure coefficients were calculated using potential flow theory as follows: $c_p = 1 - (2sin(\theta) + \frac{\Gamma}{2\pi aU})^2 = 1 - (2sin(\theta) - 2sin(\alpha))^2$. These theoretical values were then plotted along with the method of panels pressure coefficients to directly compare the two methods. Finally, I found the separation points by implementing Thwaites criteria (listed in the Formulation section). Specifically, the derivative in Thwaites criteria was computed using mkfdstencil.m, a function provided on bcourses by Eric. The integral in Thwaites criteria was computed using the trapz() built in MATLAB method.

5 Results

For the symmetric case, the Kutta condition was enforced at $\theta = 180°$, the result of which created a symmetric pressure distribution around the circular cylinder. Specifically, the leading edges and trailing edges had high pressure regions while the top and bottom surfaces were the suction surfaces, as can be seen by the c_p vs. θ plots below, thus the net lift is zero. In my code, the Thwaites boundary layer separation criteria predicted separation at $\theta \approx 103°$ and $\theta \approx -103°$ relative to the leading edge as the number of panels $\rightarrow \infty$.

For the asymmetric case, the Kutta condition was enforced at $\theta = -150°$, the result of which created an asymmetric pressure distribution that creates lift perpendicular to the orange horizon in figure 2b. The separation points were calculated to be $\theta \approx 106°$ and $\theta \approx -99°$ relative to the leading edge as the number of panels $\rightarrow \infty$.

As seen in figure 1b and figure 2b, as the number of panels increases the shape more closely approximates a true circular cylinder. Thus, the difference between the potential theory c_p calculation and the method of panels c_p calculation becomes increasingly negligible.

5.1 Symmetric

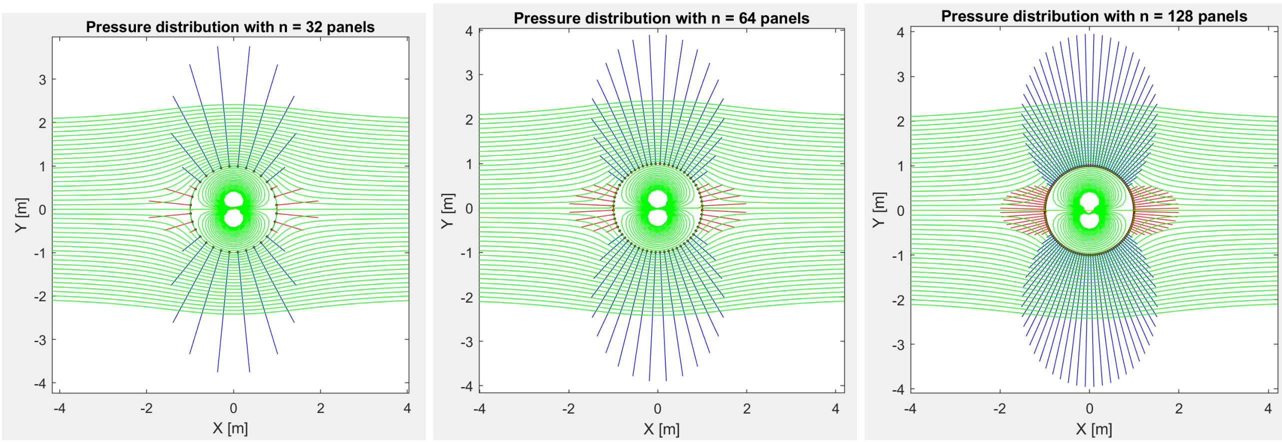

Figure 1a: Visual representation of the pressure distribution imposed on the cylinder surface. Blue lines represent negative gauge pressures, red lines are positive gauge pressures, and green lines are the analytical solutions of the streamfunction at varying levels.

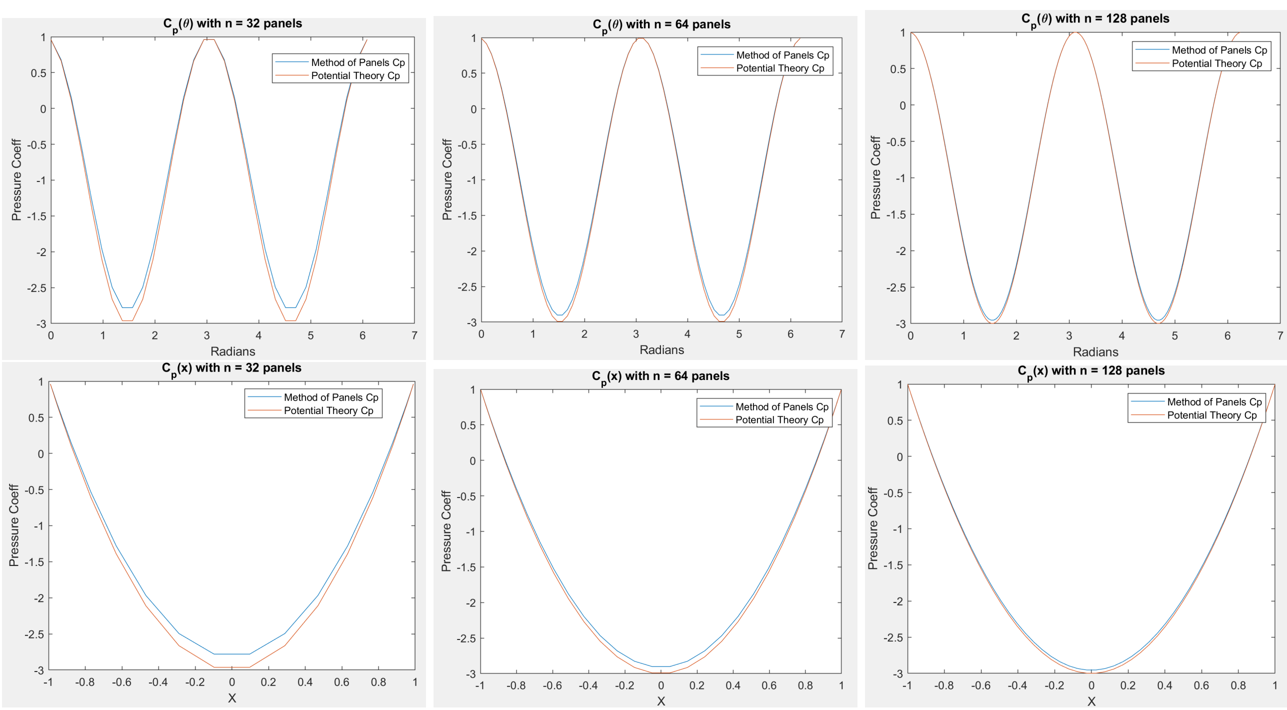

Figure 1b: Plots of c_p against radians (θ) and chord (x) as calculated by method of panels and by potential flow theory. As the panels increase, the method of panels line approaches the potential theory line.

5.2 Asymmetric

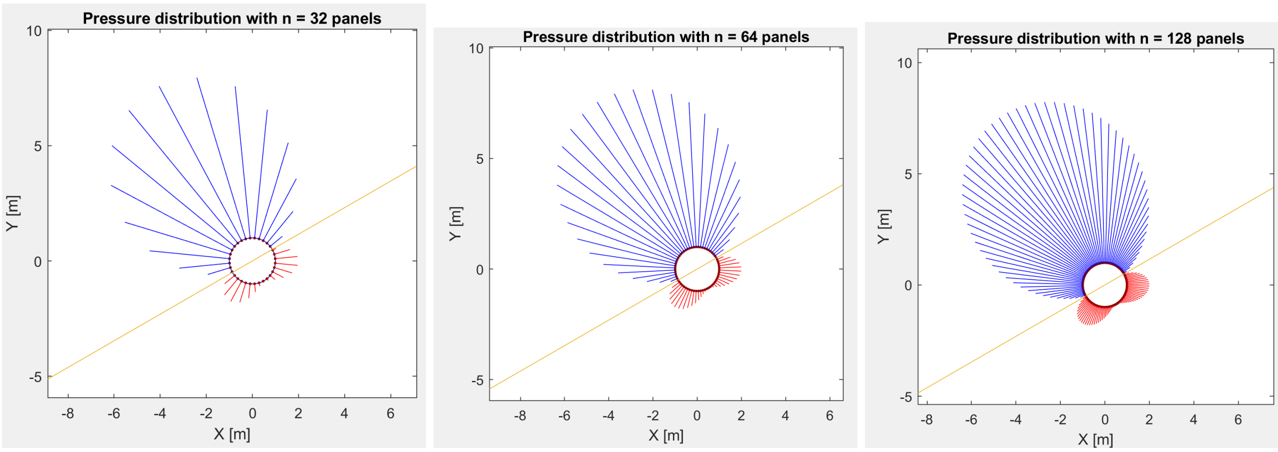

Figure 2a: Visual representation of the pressure distribution imposed on the cylinder surface. Blue lines represent negative gauge pressures and red lines represent positive gauge pressures. The leading and trailing edge are represented by the intersections with the orange horizon such that lift is perpendicular to the orange line in the direction of negative pressure.

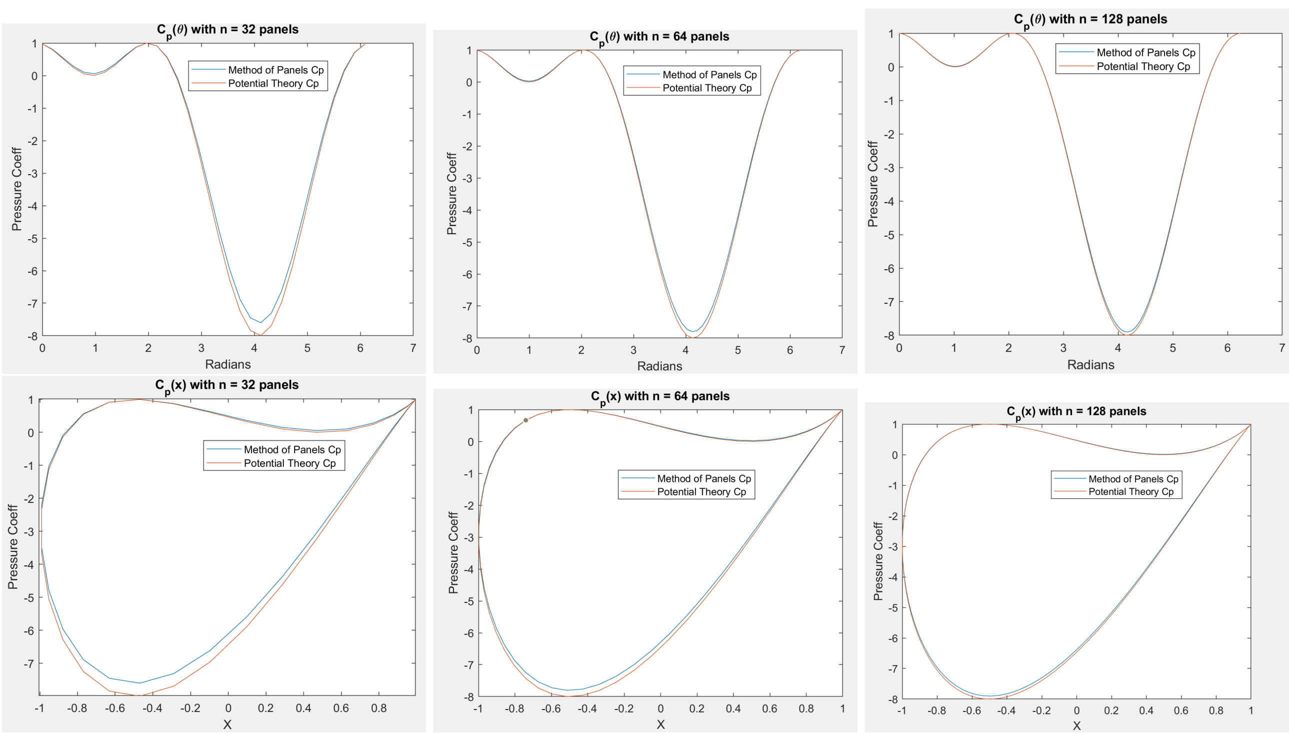

Figure 2b: Plots of c_p against radians (θ) and (x) as calculated by method of panels and by potential flow theory. As the panels increase, the method of panels line approaches the potential theory line.